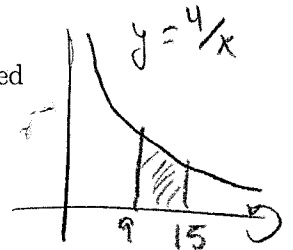


QUIZ 11 SOLUTIONS: LESSON 12
FEBRUARY 13, 2019

Write legibly, clearly indicate the question you are answering, and put a box or circle around your final answer. If you do not clearly indicate the question numbers, I will take off points. Write as much work as you need to demonstrate to me that you understand the concepts involved. If you have any questions, raise your hand and I will come over to you.

1. [5 pts] Find the volume of the solid obtained by revolving the region enclosed by the curves

$$y = \frac{4}{x}, y = 0, x = 9, \text{ and } x = 15$$

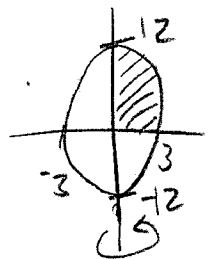


$$\begin{aligned} \text{Vol} &= \pi \int_9^{15} \left(\frac{4}{x}\right)^2 dx \quad \text{about the } x\text{-axis.} \\ &= \pi \int_9^{15} \left(\frac{16}{x^2}\right) dx \\ &= \pi \int_9^{15} 16x^{-2} dx \\ &= \frac{16\pi}{-2+1} x^{-2+1} \Big|_9^{15} \end{aligned}$$

$$\begin{aligned} &= -16\pi x^{-1} \Big|_9^{15} \\ &= -\frac{16\pi}{x} \Big|_9^{15} \\ &= \frac{-16\pi}{15} - \left(-\frac{16\pi}{9}\right) \\ &= \boxed{\frac{32\pi}{45}} \end{aligned}$$

2. [5 pts] Set up but do not solve the integral that describes the volume of the solid obtained by revolving the region in the first quadrant enclosed by

$$y = 4\sqrt{9-x^2}, y = 0, \text{ and } x = 0$$



$$\begin{aligned} &\text{about the } y\text{-axis.} \\ \Rightarrow y &= 4\sqrt{9-x^2} \\ \frac{y}{4} &= \sqrt{9-x^2} \end{aligned}$$

$$\text{Vol} = \int_0^{12} \pi \left(\sqrt{9 - \frac{y^2}{16}}\right)^2 dy$$

$$\frac{y^2}{16} = 9 - x^2$$

$$x^2 = 9 - \frac{y^2}{16}$$

$$x = \sqrt{9 - \frac{y^2}{16}}$$

$$\text{When } x = 0, y = \pm 12$$

$$\text{Note, } x^2 + \frac{y^2}{16} = 9 \Rightarrow \frac{x^2}{9} + \frac{y^2}{144} = 1$$

describes an ellipse passing through the points $(\pm 3, 0), (0, \pm 12)$